# NAG C Library Function Document

# nag_ztrevc (f08qxc)

## 1 Purpose

nag_ztrevc (f08qxc) computes selected left and/or right eigenvectors of a complex upper triangular matrix.

## 2 Specification

```
void nag_ztrevc (Nag_OrderType order, Nag_SideType side, Nag_HowManyType how_many,
      const Boolean select[], Integer n, Complex t[], Integer pdt, Complex vl[],
      Integer pdvl, Complex vr[], Integer pdvr, Integer mm, Integer *m,
      NagError *fail)
```

## 3 Description

nag_ztrevc (f08qxc) computes left and/or right eigenvectors of a complex upper triangular matrix $T$. Such a matrix arises from the Schur factorization of a complex general matrix, as computed by nag_zhseqr (f08psc), for example.

The right eigenvector $x$, and the left eigenvector $y$, corresponding to an eigenvalue $\lambda$, are defined by:

$$Tx = \lambda x \text{ and } y^H T = \lambda y^H \text{ (or } T^H y = \bar{\lambda} y).$$

The function can compute the eigenvectors corresponding to selected eigenvalues, or it can compute all the eigenvectors. In the latter case the eigenvectors may optionally be pre-multiplied by an input matrix $Q$. Normally $Q$ is a unitary matrix from the Schur factorization of a matrix $A$ as $A = QTQ^H$; if $x$ is a (left or right) eigenvector of $T$, then $Qx$ is an eigenvector of $A$.

The eigenvectors are computed by forward or backward substitution. They are scaled so that $\max(|\mathrm{Re}(x_i)| + |\mathrm{Im}(x_i)|) = 1$.

## 4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

## 5 Parameters

1: **order** – Nag_OrderType *Input*

*On entry*: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

*Constraint*: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **side** – Nag_SideType *Input*

*On entry*: indicates whether left and/or right eigenvectors are to be computed as follows:

if **side** = **Nag_RightSide**, only right eigenvectors are computed;

if **side** = **Nag_LeftSide**, only left eigenvectors are computed;

if **side** = **Nag_BothSides**, both left and right eigenvectors are computed.

*Constraint*: **side** = **Nag_RightSide**, **Nag_LeftSide** or **Nag_BothSides**.

3: **how_many** – Nag_HowManyType *Input*

*On entry*: indicates how many eigenvectors are to be computed as follows:

if **how_many** = **Nag_ComputeAll**, all eigenvectors (as specified by **side**) are computed;

if **how_many** = **Nag_BackTransform**, all eigenvectors (as specified by **side**) are computed and then pre-multiplied by the matrix $Q$ (which is overwritten);

if **how_many** = **Nag_ComputeSelected**, selected eigenvectors (as specified by **side** and **select**) are computed.

*Constraint*: **how_many** = **Nag_ComputeAll**, **Nag_BackTransform** or **Nag_ComputeSelected**.

4: **select**[$dim$] – const Boolean *Input*

**Note:** the dimension, $dim$, of the array **select** must be at least $\max(1, \mathbf{n})$ when **how_many** = **Nag_ComputeSelected** and at least 1 otherwise.

*On entry*: **select** specifies which eigenvectors are to be computed if **how_many** = **Nag_ComputeSelected**. To obtain the eigenvector corresponding to the eigenvalue $\lambda_j$, **select**[$j$] must be set **TRUE**.

**select** is not referenced if **how_many** = **Nag_ComputeAll** or **Nag_BackTransform**.

5: **n** – Integer *Input*

*On entry*: $n$, the order of the matrix $T$.

*Constraint*: $\mathbf{n} \geq 0$.

6: **t**[$dim$] – Complex *Input/Output*

**Note:** the dimension, $dim$, of the array **t** must be at least $\max(1, \mathbf{pdt} \times \mathbf{n})$.

If **order** = **Nag_ColMajor**, the $(i, j)$th element of the matrix $T$ is stored in $\mathbf{t}[(j-1) \times \mathbf{pdt} + i - 1]$ and if **order** = **Nag_RowMajor**, the $(i, j)$th element of the matrix $T$ is stored in $\mathbf{t}[(i-1) \times \mathbf{pdt} + j - 1]$.

*On entry*: the $n$ by $n$ upper triangular matrix $T$, as returned by nag_zhseqr (f08psc).

*On exit*: **t** is used as internal workspace prior to being restored and hence is unchanged.

7: **pdt** – Integer *Input*

*On entry*: the stride separating matrix row or column elements (depending on the value of **order**) in the array **t**.

*Constraint*: $\mathbf{pdt} \geq \max(1, \mathbf{n})$.

8: **vl**[$dim$] – Complex *Input/Output*

**Note:** the dimension, $dim$, of the array **vl** must be at least

$\max(1, \mathbf{pdvl} \times \mathbf{mm})$ when **side** = **Nag_LeftSide** or **Nag_BothSides** and **order** = **Nag_ColMajor**;

$\max(1, \mathbf{pdvl} \times \mathbf{n})$ when **side** = **Nag_LeftSide** or **Nag_BothSides** and **order** = **Nag_RowMajor**;

1 when **side** = **Nag_RightSide**.

If **order** = **Nag_ColMajor**, the $(i, j)$th element of the matrix is stored in $\mathbf{vl}[(j-1) \times \mathbf{pdvl} + i - 1]$ and if **order** = **Nag_RowMajor**, the $(i, j)$th element of the matrix is stored in $\mathbf{vl}[(i-1) \times \mathbf{pdvl} + j - 1]$.

*On entry*: if **how_many** = **Nag_BackTransform** and **side** = **Nag_LeftSide** or **Nag_BothSides**, **vl** must contain an $n$ by $n$ matrix $Q$ (usually the matrix of Schur vectors returned by nag_zhseqr (f08psc)). If **how_many** = **Nag_ComputeAll** or **Nag_ComputeSelected**, **vl** need not be set.

*On exit*: if **side** = **Nag_LeftSide** or **Nag_BothSides**, **vl** contains the computed left eigenvectors (as specified by **how_many** and **select**). The eigenvectors are stored consecutively in the rows or columns (depending on the value of **order**) of the array, in the same order as their eigenvalues.

**vl** is not referenced if **side** = **Nag_RightSide**.

9: **pdvl** – Integer *Input*

*On entry*: the stride separating matrix row or column elements (depending on the value of **order**) in the array **vl**.

*Constraints*:

if **order** = **Nag_ColMajor**,
  if **side** = **Nag_LeftSide** or **Nag_BothSides**, **pdvl** $\geq \max(1, \mathbf{n})$;
  if **side** = **Nag_RightSide**, **pdvl** $\geq 1$;

if **order** = **Nag_RowMajor**,
  if **side** = **Nag_LeftSide** or **Nag_BothSides**, **pdvl** $\geq \max(1, \mathbf{mm})$;
  if **side** = **Nag_RightSide**, **pdvl** $\geq 1$.

10: **vr**[$dim$] – Complex *Input/Output*

**Note:** the dimension, $dim$, of the array **vr** must be at least

$\max(1, \mathbf{pdvr} \times \mathbf{mm})$ when **side** = **Nag_RightSide** or **Nag_BothSides** and **order** = **Nag_ColMajor**;

$\max(1, \mathbf{pdvr} \times \mathbf{n})$ when **side** = **Nag_RightSide** or **Nag_BothSides** and **order** = **Nag_RowMajor**;

1 when **side** = **Nag_LeftSide**.

If **order** = **Nag_ColMajor**, the $(i, j)$th element of the matrix is stored in **vr**$[(j-1) \times \mathbf{pdvr} + i - 1]$ and if **order** = **Nag_RowMajor**, the $(i, j)$th element of the matrix is stored in **vr**$[(i-1) \times \mathbf{pdvr} + j - 1]$.

*On entry*: if **how_many** = **Nag_BackTransform** and **side** = **Nag_RightSide** or **Nag_BothSides**, **vr** must contain an $n$ by $n$ matrix $Q$ (usually the matrix of Schur vectors returned by nag_zhseqr (f08psc)). If **how_many** = **Nag_ComputeAll** or **Nag_ComputeSelected**, **vr** need not be set.

*On exit*: if **side** = **Nag_RightSide** or **Nag_BothSides**, **vr** contains the computed right eigenvectors (as specified by **how_many** and **select**). The eigenvectors are stored consecutively in the rows or columns (depending on the value of **order**) of the array, in the same order as their eigenvalues.

**vr** is not referenced if **side** = **Nag_LeftSide**.

11: **pdvr** – Integer *Input*

*On entry*: the stride separating matrix row or column elements (depending on the value of **order**) in the array **vr**.

*Constraints*:

if **order** = **Nag_ColMajor**,
  if **side** = **Nag_RightSide** or **Nag_BothSides**, **pdvr** $\geq \max(1, \mathbf{n})$;
  if **side** = **Nag_LeftSide**, **pdvr** $\geq 1$;

if **order** = **Nag_RowMajor**,
  if **side** = **Nag_RightSide** or **Nag_BothSides**, **pdvr** $\geq \max(1, \mathbf{mm})$;
  if **side** = **Nag_LeftSide**, **pdvr** $\geq 1$.

12: **mm** – Integer *Input*

*On entry*: the number of rows or columns (depending on the value of **order**) in the arrays **vl** and/or **vr**. The precise number of rows or columns required, $required\_rowcol$, is $n$ if **how_many** = **Nag_ComputeAll** or **Nag_BackTransform**; if **how_many** = **Nag_ComputeSelected**, $required\_rowcol$ is the number of selected eigenvectors (see **select**), in which case $0 \leq required\_rowcol \leq n$.

*Constraint*: **mm** $\geq required\_rowcol$.

13: **m** – Integer * *Output*

*On exit*: $required\_rowcol$, the number of selected eigenvectors. If **how_many** = **Nag_ComputeAll** or **Nag_BackTransform**, **m** is set to $n$.

14: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

**NE_INT**

On entry, **n** = ⟨*value*⟩.
Constraint: **n** $\geq 0$.

On entry, **mm** = ⟨*value*⟩.
Constraint: **mm** $\geq required\_rowcol$, where $required\_rowcol$ is the number of selected eigenvectors.

On entry, **pdt** = ⟨*value*⟩.
Constraint: **pdt** $> 0$.

On entry, **pdvl** = ⟨*value*⟩.
Constraint: **pdvl** $> 0$.

On entry, **pdvr** = ⟨*value*⟩.
Constraint: **pdvr** $> 0$.

**NE_INT_2**

On entry, **pdt** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: **pdt** $\geq \max(1, \mathbf{n})$.

**NE_ENUM_INT_2**

On entry, **side** = ⟨*value*⟩, **n** = ⟨*value*⟩, **pdvl** = ⟨*value*⟩.
Constraint: if **side** = **Nag_LeftSide** or **Nag_BothSides**, **pdvl** $\geq \max(1, \mathbf{n})$;
if **side** = **Nag_RightSide**, **pdvl** $\geq 1$.

On entry, **side** = ⟨*value*⟩, **n** = ⟨*value*⟩, **pdvr** = ⟨*value*⟩.
Constraint: if **side** = **Nag_RightSide** or **Nag_BothSides**, **pdvr** $\geq \max(1, \mathbf{n})$;
if **side** = **Nag_LeftSide**, **pdvr** $\geq 1$.

On entry, **side** = ⟨*value*⟩, **mm** = ⟨*value*⟩, **pdvl** = ⟨*value*⟩.
Constraint: if **side** = **Nag_LeftSide** or **Nag_BothSides**, **pdvl** $\geq \max(1, \mathbf{mm})$;
if **side** = **Nag_RightSide**, **pdvl** $\geq 1$.

On entry, **side** = ⟨*value*⟩, **mm** = ⟨*value*⟩, **pdvr** = ⟨*value*⟩.
Constraint: if **side** = **Nag_RightSide** or **Nag_BothSides**, **pdvr** $\geq \max(1, \mathbf{mm})$;
if **side** = **Nag_LeftSide**, **pdvr** $\geq 1$.

**NE_ALLOC_FAIL**

Memory allocation failed.

**NE_BAD_PARAM**

On entry, parameter ⟨*value*⟩ had an illegal value.

**NE_INTERNAL_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

If $x_i$ is an exact right eigenvector, and $\tilde{x}_i$ is the corresponding computed eigenvector, then the angle $\theta(\tilde{x}_i, x_i)$ between them is bounded as follows:

$$\theta(\tilde{x}_i, x_i) \leq \frac{c(n)\epsilon\|T\|_2}{sep_i}$$

where $sep_i$ is the reciprocal condition number of $x_i$.

The condition number $sep_i$ may be computed by calling nag_ztrsna (f08qyc).

## 8 Further Comments

The real analogue of this function is nag_dtrevc (f08qkc).

## 9 Example

See Section 9 of the document for nag_zgebal (f08nvc).